

**CORRIGENDUM TO THE PAPER, “A NEW ITERATION
PROCESS FOR APPROXIMATION OF COMMON FIXED
POINTS FOR FINITE FAMILIES OF TOTAL ASYMTOTICALLY
NONEXPANSIVE MAPPINGS”.**

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ABSTRACT. A gap in the proof of Theorem 3.5 in the above paper is observed. The argument used on page 11, starting from line 8 from bottom to the end of the proof of the theorem is not correct. In this corrigendum, it is our aim to close this gap.

1. INTRODUCTION

Let K be a nonempty subset of a real normed space E . A mapping $T : K \rightarrow K$ is said to be *nonexpansive* if $\|Tx - Ty\| \leq \|x - y\|$ for all $x, y \in K$.

The mapping T is called *asymptotically nonexpansive* if there exists a sequence $\{\mu_n\}_{n \geq 1} \subset [0, \infty)$ with $\lim_{n \rightarrow \infty} \mu_n = 0$ such that for all $x, y \in K$,

$$\|T^n x - T^n y\| \leq (1 + \mu_n)\|x - y\| \text{ for all } n \geq 1;$$

and T is said to be uniformly L -Lipschitzian if there exists a constant $L \geq 0$ such that

$$\|T^n x - T^n y\| \leq L\|x - y\| \quad \forall x, y \in K.$$

The class of asymptotically nonexpansive mappings was introduced by Goebel and Kirk [5] as a generalization of the class of nonexpansive mappings. They proved that if K is a nonempty closed convex bounded subset of a uniformly convex real Banach space and T is an asymptotically nonexpansive self-mapping of K , then T has a fixed point.

A mapping T is said to be *asymptotically nonexpansive in the intermediate sense* (see e.g., [2]) if it is continuous and the following inequality holds:

$$(1) \quad \limsup_{n \rightarrow \infty} \sup_{x, y \in K} (\|T^n x - T^n y\| - \|x - y\|) \leq 0.$$

Observe that if we define

$$a_n := \sup_{x, y \in K} (\|T^n x - T^n y\| - \|x - y\|), \text{ and } \sigma_n = \max\{0, a_n\},$$

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then $\sigma_n \rightarrow 0$ as $n \rightarrow \infty$ and (1) reduces to

$$(2) \quad \|T^n x - T^n y\| \leq \|x - y\| + \sigma_n, \text{ for all } x, y \in K, n \geq 1.$$

The class of mappings which are asymptotically nonexpansive in the intermediate sense was introduced by Bruck *et al.* [2]. It is known [7] that if K is a nonempty closed convex bounded subset of a uniformly convex real Banach space E and T is a self-mapping of K which is asymptotically nonexpansive in the intermediate sense, then T has a fixed point. It is worth mentioning that the class of mappings which are asymptotically nonexpansive in the intermediate sense contains properly the class of asymptotically nonexpansive mappings (see, e.g., [6]).

Sahu [9], introduced the class of nearly Lipschitzian mappings. Let K be a nonempty subset of a normed space E and let $\{a_n\}_{n \geq 1}$ be a sequence in $[0, +\infty)$ such that $\lim_{n \rightarrow \infty} a_n = 0$. A mapping $T : K \rightarrow K$ is called *nearly Lipschitzian* with respect to $\{a_n\}_{n \geq 1}$ if for each $n \in \mathbb{N}$, there exists $k_n \geq 0$ such that

$$(3) \quad \|T^n x - T^n y\| \leq k_n(\|x - y\| + a_n) \quad \forall x, y \in K.$$

Define

$$\eta(T^n) := \sup \left\{ \frac{\|T^n x - T^n y\|}{\|x - y\| + a_n} : x, y \in K, x \neq y \right\}.$$

Observe that for any sequence $\{k_n\}_{n \geq 1}$ satisfying (3), $\eta(T^n) \leq k_n \quad \forall n \in \mathbb{N}$ and that

$$\|T^n x - T^n y\| \leq \eta(T^n)(\|x - y\| + a_n) \quad \forall x, y \in K, n \in \mathbb{N}.$$

$\eta(T^n)$ is called the *nearly Lipschitz constant* of the mapping T . A nearly Lipschitzian mapping T is said to be

- *nearly contraction* if $\eta(T^n) < 1$ for all $n \in \mathbb{N}$;
- *nearly nonexpansive* if $\eta(T^n) = 1$ for all $n \in \mathbb{N}$;
- *nearly asymptotically nonexpansive* if $\eta(T^n) \geq 1$ for all $n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} \eta(T^n) = 1$;
- *nearly uniform L -Lipschitzian* if $\eta(T^n) \leq L$ for all $n \in \mathbb{N}$;
- *nearly uniform k -contraction* if $\eta(T^n) \leq k < 1$ for all $n \in \mathbb{N}$.

Example 1. (See Sahu [9]) Let $E = \mathbb{R}$, $K = [0, 1]$. Define $T : K \rightarrow K$ by

$$Tx = \begin{cases} \frac{1}{2}, & \text{if } x \in [0, \frac{1}{2}] \\ 0, & \text{if } x \in (\frac{1}{2}, 1]. \end{cases}$$

It is obvious that T is not continuous, and thus, not Lipschitz. However, T is nearly nonexpansive. In fact, for a real sequence $\{a_n\}_{n \geq 1}$ with $a_1 = \frac{1}{2}$ and $a_n \rightarrow 0$ as $n \rightarrow \infty$, we have

$$\|Tx - Ty\| \leq \|x - y\| + a_1 \quad \forall x, y \in K$$

and

$$\|T^n x - T^n y\| \leq \|x - y\| + a_n \quad \forall x, y \in K, n \geq 2.$$

This is because $T^n x = \frac{1}{2} \quad \forall x \in [0, 1], n \geq 2$.

Remark 2. If K is a bounded domain of an asymptotically nonexpansive mapping T , then T is nearly nonexpansive. In fact, for all $x, y \in K$ and $n \in \mathbb{N}$, we have

$$\|T^n x - T^n y\| \leq (1 + \mu_n)\|x - y\| \leq \|x - y\| + \text{diam}(K)\mu_n.$$

Furthermore, we easily observe that every nearly nonexpansive mapping is nearly asymptotically nonexpansive with $\eta(T^n) = 1 \quad \forall n \in \mathbb{N}$.

Remark 3. If K is a bounded domain of a nearly asymptotically nonexpansive mapping T , then T is asymptotically nonexpansive in the intermediate sense. To see this, let T be a nearly asymptotically nonexpansive mapping. Then,

$$\|T^n x - T^n y\| \leq \eta(T^n)(\|x - y\| + a_n) \quad \forall x, y \in K, \quad n \geq 1,$$

which implies that

$$\sup_{x, y \in K} (\|T^n x - T^n y\| - \|x - y\|) \leq (\eta(T^n) - 1) \text{diam}(K) + \eta(T^n) a_n, \quad n \geq 1.$$

Hence,

$$\limsup_{n \rightarrow \infty} \sup_{x, y \in K} (\|T^n x - T^n y\| - \|x - y\|) \leq 0.$$

We observe from Remarks 2 and 3 that the class of nearly nonexpansive mappings and nearly asymptotically nonexpansive mappings are intermediate classes between the class of asymptotically nonexpansive mappings and that of asymptotically nonexpansive in the intermediate sense mappings.

Alber *et al.* [1] introduced a more general class of asymptotically nonexpansive mappings called *total asymptotically nonexpansive mappings* and studied methods of approximation of fixed points of mappings belonging to this class.

Definition 4. A mapping $T : K \rightarrow K$ is said to be *total asymptotically nonexpansive* if there exist nonnegative real sequences $\{\mu_n\}$ and $\{l_n\}$, $n \geq 1$ with $\mu_n, l_n \rightarrow 0$ as $n \rightarrow \infty$ and strictly increasing continuous function $\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ with $\phi(0) = 0$ such that for all $x, y \in K$,

$$(4) \quad \|T^n x - T^n y\| \leq \|x - y\| + \mu_n \phi(\|x - y\|) + l_n, \quad n \geq 1.$$

Remark 5. If $\phi(\lambda) = \lambda$, then (4) reduces to

$$\|T^n x - T^n y\| \leq (1 + \mu_n) \|x - y\| + l_n, \quad n \geq 1.$$

In addition, if $l_n = 0$ for all $n \geq 1$, then total asymptotically nonexpansive mappings coincide with asymptotically nonexpansive mappings. If $\mu_n = 0$ and $l_n = 0$ for all $n \geq 1$, we obtain from (4) the class of mappings that includes the class of nonexpansive mappings. If $\mu_n = 0$ and $l_n = \sigma_n = \max\{0, a_n\}$, where $a_n := \sup_{x, y \in K} (\|T^n x - T^n y\| - \|x - y\|)$ for all $n \geq 1$, then (4) reduces to (2) which has been studied as mappings which are asymptotically nonexpansive in the intermediate sense.

Remark 6. The idea of Definition 4 is to unify various definitions of classes of mappings associated with the class of asymptotically nonexpansive mappings and to prove a general convergence theorems applicable to all these classes of nonlinear mappings.

This work is motivated by the recent paper of Chidume and Ofoedu [4]. A gap in the proof of Theorem 3.5 in [4] is observed. The argument used on page 11, starting from line 8 from bottom to the end of the proof of the theorem is not correct. In this corrigendum, it is our aim to close this gap.

2. PRELIMINARY

In the sequel, we shall need the following

Lemma 7. *Let $\{a_n\}$, $\{\alpha_n\}$ and $\{b_n\}$ be sequences of nonnegative real numbers such that*

$$a_{n+1} \leq (1 + \alpha_n)a_n + b_n.$$

Suppose that $\sum_{n=1}^{\infty} \alpha_n < \infty$ and $\sum_{n=1}^{\infty} b_n < \infty$. Then $\{a_n\}$ is bounded and $\lim_{n \rightarrow \infty} a_n$ exists. Moreover, if in addition, $\liminf_{n \rightarrow \infty} a_n = 0$, then $\lim_{n \rightarrow \infty} a_n = 0$.

Lemmas:

Lemma 8. [11] *Let E be a uniformly convex Banach space and $B_R(0)$ be a closed ball of E . Then there exists a continuous strictly increasing convex function $g : [0, \infty) \rightarrow [0, \infty)$ with $g(0) = 0$ such that*

$$\|\alpha_0 x_0 + \alpha_1 x_1 + \alpha_2 x_2 + \dots + \alpha_r x_r\|^2 \leq \sum_{i=0}^r \alpha_i \|x_i\|^2 - \alpha_s \alpha_t g(\|x_s - x_t\|),$$

for any $s, t \in \{0, 1, 2, \dots, r\}$ and for $x_i \in B_R(0) := \{x \in E : \|x\| \leq R\}$, $i = 0, 1, 2, \dots, r$ with $\sum_{i=0}^r \alpha_i = 1$.

3. MAIN RESULTS

Lemma 9. *Let E be a real Banach space, K be a nonempty closed convex subset of E and $T : K \rightarrow K$ be a total asymptotically nonexpansive mappings with sequences $\{\mu_n\}$, $\{\ell_n\}$ $n \geq 1$ Suppose that there exist M , $M^* > 0$ such that $\phi(\lambda) \leq M^* \lambda$ for all $\lambda \geq M$ then*

$$\|T^n x - T^n y\| \leq (1 + \mu_n M^*) \|x - y\| + \mu_n \phi(M) + \ell_n \quad \forall x, y \in K, \quad \forall n \geq 1.$$

Proof. Since $\phi : [0, +\infty) \rightarrow [0, +\infty)$ is strictly increasing continuous function, we have that $\lambda \leq M$ implies $\phi(\lambda) \leq \phi(M)$; and by the hypothesis, $\phi(\lambda) \leq M^* \lambda$ for all $\lambda \geq M$. It therefore not difficult to see that $\phi(\lambda) \leq \phi(M) + M^* \lambda$. Using this and the fact that T is total asymptotically nonexpansive, we obtain the required inequality. \square

Let K be a nonempty closed convex subset of a real normed space E . Let $T_1, T_2, \dots, T_m : K \rightarrow K$ be m total asymptotically nonexpansive mappings. We first note that the recursion formula (3.1) on page 8 of [4] contains a typo. The correct formula is

$$(5) \quad x_1 \in K, \quad x_{n+1} = \alpha_{0n} x_n + \sum_{i=1}^m \alpha_{in} T_i^n x_n, \quad n \geq 1,$$

where $\{\alpha_{in}\}_{n \geq 1}$, $i = 0, 1, 2, \dots, m$ are sequences in (γ_1, γ_2) , for some $\gamma_1, \gamma_2 \in (0, 1)$ such that $\sum_{i=0}^m \alpha_{in} = 1$.

Theorem 10. (Theorem 3.1 of [4]) *Let E be a real Banach space, K be a nonempty closed convex subset of E and $T_i : K \rightarrow K$, $i = 1, 2, \dots, m$ be m total asymptotically nonexpansive mappings with sequences $\{\mu_{in}\}$, $\{\ell_{in}\}$ $n \geq 1$, $i = 1, 2, \dots, m$ such that*

$F := \cap_{i=1}^m F(T_i) \neq \emptyset$. Let $\{x_n\}$ be given by (5). Suppose $\sum_{n=1}^{\infty} \mu_{in} < \infty$, $\sum_{n=1}^{\infty} l_{in} < \infty$ for $i = 1, 2, \dots, m$ and suppose that there exist M_i , $M_i^* > 0$ such that $\phi_i(\lambda_i) \leq M_i^* \lambda_i$ for all $\lambda_i \geq M_i$, $i = 1, 2, \dots, m$, then the sequence $\{x_n\}_{n \geq 1}$ is bounded and $\lim_{n \rightarrow \infty} \|x_n - p\|$ exists, $p \in F$.

Proof. The proof is exactly the proof of Theorem 3.1 of [4]. \square

We now restate and give an alternative proof of Theorem 3.5 of [4].

Theorem 11. (Correctedd version of Theorem 3.5 of [4]) Let E be a uniformly convex real Banach space, K be a nonempty closed convex subset of E and $T_i : K \rightarrow K$, $i = 1, 2, \dots, m$ be m total asymptotically nonexpansive mappings with sequences $\{\mu_{in}\}$, $\{l_{in}\} \subset [0, \infty)$ such that $\sum_{n=1}^{\infty} \mu_{in} < \infty$, $\sum_{n=1}^{\infty} l_{in} < \infty$, $i = 1, 2, \dots, m$ and $F := \bigcap_{i=1}^m F(T_i) \neq \emptyset$. From arbitrary $x_1 \in E$, define the sequence $\{x_n\}$ by (5). Suppose that there exist M_i , $M_i^* > 0$ such that $\phi_i(\lambda_i) \leq M_i^* \lambda_i$ whenever $\lambda_i \geq M_i$, $i = 1, 2, \dots, m$, then $\lim_{n \rightarrow \infty} \|x_n - T_i^n x_n\| = 0$, $i = 1, 2, \dots, m$.

Proof. Let $p \in F$, then using the recursion formula (5) and Lemma 8, we have (for any $j \in \{1, 2, \dots, m\}$) that

$$\begin{aligned}
 \|x_{n+1} - p\|^2 &= \left\| \alpha_{0n} x_n + \sum_{i=1}^m \alpha_{in} T_i^n x_n - p \right\|^2 = \left\| \alpha_{0n} (x_n - p) + \sum_{i=1}^m \alpha_{in} (T_i^n x_n - p) \right\|^2 \\
 &\leq \alpha_{0n} \|x_n - p\|^2 + \sum_{i=1}^m \alpha_{in} \|T_i^n x_n - p\|^2 - \alpha_{0n} \alpha_{jn} g(\|x_n - T_j^n x_n\|) \\
 &\leq \alpha_{0n} \|x_n - p\|^2 + \sum_{i=1}^m \alpha_{in} \left(\|x_n - p\| + \mu_{in} \phi_i(M_i) \right. \\
 &\quad \left. + \mu_{in} M_i^* \|x_n - p\| + l_{in} \right)^2 - \alpha_{0n} \alpha_{jn} g(\|x_n - T_j^n x_n\|) \\
 &= \alpha_{0n} \|x_n - p\|^2 + \sum_{i=1}^m \alpha_{in} \|x_n - p\|^2 \\
 &\quad + \sum_{i=1}^m \alpha_{in} \left(2\|x_n - p\| [\mu_{in} \phi_i(M_i) + \mu_{in} M_i^* \|x_n - p\| + l_{in}] \right. \\
 &\quad \left. + [\mu_{in} \phi_i(M_i) + \mu_{in} M_i^* \|x_n - p\| + l_{in}]^2 \right) - \alpha_{0n} \alpha_{jn} g(\|x_n - T_j^n x_n\|) \\
 &= \|x_n - p\|^2 + \sum_{i=1}^m \alpha_{in} \left(2\|x_n - p\| [\mu_{in} \phi_i(M_i) + \mu_{in} M_i^* \|x_n - p\| + l_{in}] \right. \\
 &\quad \left. + [\mu_{in} \phi_i(M_i) + \mu_{in} M_i^* \|x_n - p\| + l_{in}]^2 \right) - \alpha_{0n} \alpha_{jn} g(\|x_n - T_j^n x_n\|).
 \end{aligned}
 \tag{6}$$

So, since $\alpha_{in} \in (\gamma_1, \gamma_2)$, $i = 0, 1, 2, \dots, m$, we obtain from (6) that

$$\begin{aligned}
 \gamma_1^2 g(\|x_n - T_j^n x_n\|) &\leq \alpha_{0n} \alpha_{jn} g(\|x_n - T_j^n x_n\|) \\
 &\leq \|x_n - p\|^2 - \|x_{n+1} - p\|^2 + \sum_{i=1}^m \alpha_{in} \left(2\|x_n - p\| [\mu_{in} \phi_i(M_i) \right. \\
 &\quad \left. + \mu_{in} M_i^* \|x_n - p\| + l_{in}] + [\mu_{in} \phi_i(M_i) + \mu_{in} M_i^* \|x_n - p\| + l_{in}]^2 \right) \\
 &\leq \|x_n - p\|^2 - \|x_{n+1} - p\|^2 + \gamma_2 \sum_{i=1}^m \left(2\|x_n - p\| [\mu_{in} \phi_i(M_i) \right. \\
 &\quad \left. + \mu_{in} M_i^* \|x_n - p\| + l_{in}] + [\mu_{in} \phi_i(M_i) + \mu_{in} M_i^* \|x_n - p\| + l_{in}]^2 \right).
 \end{aligned} \tag{7}$$

But $\mu_{in} \rightarrow 0$ and $\ell_{in} \rightarrow 0$ as $n \rightarrow \infty$, $i = 1, 2, \dots, m$ and by Theorem 10, $\lim_{n \rightarrow \infty} \|x_n - p\|$ exists. Thus, since the summation in (7) is a summation of finite terms, we obtain from (7) that

$$\lim_{n \rightarrow \infty} g(\|T_j^n x_n - x_n\|) = 0 \quad \forall j \in \{1, 2, \dots, m\}.$$

So, properties of the function g (see Lemma 8) imply that

$$(8) \quad \lim_{n \rightarrow \infty} \|T_i^n x_n - x_n\| = 0, \quad i = 1, 2, \dots, m.$$

This completes the proof. \square

Theorem 12. (Corrected version of Theorem 3.6 of [4]) Let E be a uniformly convex real Banach space, K be a nonempty closed convex subset of E and $T_i : K \rightarrow K$, $i = 1, 2, \dots, m$ be m continuous total asymptotically nonexpansive mappings with sequences $\{\mu_{in}\}$, $\{l_{in}\} \subset [0, \infty)$ such that $\sum_{n=1}^{\infty} \mu_{in} < \infty$, $\sum_{n=1}^{\infty} l_{in} < \infty$, $i = 1, 2, \dots, m$

and $F := \bigcap_{i=1}^m F(T_i) \neq \emptyset$. From arbitrary $x_1 \in E$, define the sequence $\{x_n\}$ by

(5). Suppose that there exist M_i , $M_i^* > 0$ such that $\phi_i(\lambda_i) \leq M_i^* \lambda_i$ whenever $\lambda_i \geq M_i$, $i = 1, 2, \dots, m$; and that one of T_1, T_2, \dots, T_m is compact, then $\{x_n\}$ converges strongly to some $p \in F$.

Proof. Observe that from the recursion formula (5),

$$\begin{aligned}
 \|x_{n+1} - x_n\| &= \left\| \alpha_{0n} x_n + \sum_{i=1}^m \alpha_{in} T_i^n x_n - x_n \right\| \\
 (9) \quad &\leq \sum_{i=1}^m \alpha_{in} \|T_i^n x_n - x_n\| \leq \gamma_2 \sum_{i=1}^m \|T_i^n x_n - x_n\|.
 \end{aligned}$$

Hence, using (8) and (9), we obtain that

$$(10) \quad \lim_{n \rightarrow \infty} \|x_{n+1} - x_n\| = 0.$$

Without loss of generality, let T_1 be compact. Since T_1 is continuous and compact, it is completely continuous. Thus, there exists a subsequence $\{T_1^{n_k} x_{n_k}\}$ of $\{T_1^n x_n\}$ such that $T_1^{n_k} x_{n_k} \rightarrow x^*$ as $k \rightarrow \infty$ for some $x^* \in E$. Thus $T_1^{n_k+1} x_{n_k} \rightarrow T_1 x^*$ as $k \rightarrow \infty$. Furthermore, (8) and the fact that $T_1^{n_k} x_{n_k} \rightarrow x^*$ as $k \rightarrow \infty$ imply that $\lim_{k \rightarrow \infty} x_{n_k} = x^*$. Also from (8) $T_2^{n_k} x_{n_k} \rightarrow x^*$, $T_3^{n_k} x_{n_k} \rightarrow x^*$, ..., $T_m^{n_k} x_{n_k} \rightarrow x^*$

as $k \rightarrow \infty$. Thus, $T_2^{n_k+1}x_{n_k} \rightarrow T_2x^*$, $T_3^{n_k+1}x_{n_k} \rightarrow T_3x^*$, ..., $T_m^{n_k+1}x_{n_k} \rightarrow T_mx^*$ as $k \rightarrow \infty$. Now, since from (10), $\|x_{n_k+1} - x_{n_k}\| \rightarrow 0$ as $k \rightarrow \infty$, it follows that $x_{n_k+1} \rightarrow x^*$ as $k \rightarrow \infty$. Next, we show that $x^* \in F$. Observe that

$$(11) \quad \begin{aligned} \|x^* - T_i x^*\| &\leq \|x^* - x_{n_k+1}\| + \|x_{n_k+1} - T_i^{n_k+1} x_{n_k+1}\| \\ &\quad + \|T_i^{n_k+1} x_{n_k+1} - T_i^{n_k+1} x_{n_k}\| + \|T_i^{n_k+1} x_{n_k} - T_i x^*\|. \end{aligned}$$

Taking limit as $k \rightarrow \infty$ in (11) (using Lemma 9, (8) and (10)), we have that $x^* = T_i x^*$ ($i = 1, 2, \dots, m$) and so $x^* \in F(T_i)$ ($i = 1, 2, \dots, m$). But by Theorem 10, $\lim_{n \rightarrow \infty} \|x_n - p\|$ exists, $p \in F$. Hence, $\{x_n\}$ converges strongly to $x^* \in F$. This completes the proof. \square

Remark 13. We note that the argument (due to the fact that T is uniformly continuous) $\lim_{n \rightarrow \infty} \|x_{n+1} - x_n\| = 0$ implies $\lim_{n \rightarrow 0} \|T^{n+1}x_{n+1} - T^{n+1}x_n\| = 0$ as used in [3], [4] and [10] is not always true. To see this, consider \mathbb{R} , the set of real numbers endowed with the usual topology and the mapping $T : \mathbb{R} \rightarrow \mathbb{R}$ defined by $Tx = 3x$ for all $x \in \mathbb{R}$. It is clear that T is uniformly continuous. Now, let $\{x_n\}_{n \geq 1}$ in \mathbb{R} be a sequence defined by $x_n = 1 + \frac{1}{n} \forall n \geq 1$. We can easily see that $\lim_{n \rightarrow \infty} |x_{n+1} - x_n| = 0$ but

$$\begin{aligned} |T^{n+1}x_{n+1} - T^{n+1}x_n| &= \left| \left(3^{n+1} + \frac{3^{n+1}}{n+1} \right) - \left(3^{n+1} + \frac{3^{n+1}}{n} \right) \right| \\ &= \left| \frac{3^{n+1}}{n+1} - \frac{3^{n+1}}{n} \right| = \frac{3^{n+1}}{n(n+1)} \rightarrow \infty \text{ as } n \rightarrow \infty. \end{aligned}$$

This follows from the fact that if we define $g : (0, +\infty) \rightarrow (0, +\infty)$ by $g(x) = \frac{3^{x+1}}{x(x+1)}$, then by L'Hospital's rule we obtain that

$$\lim_{x \rightarrow \infty} g(x) = \lim_{x \rightarrow \infty} \frac{3^{x+1}}{x^2 + x} = 3 \ln 3 \lim_{x \rightarrow \infty} \frac{3^x}{2x + 1} = \frac{3(\ln 3)^2}{2} \lim_{x \rightarrow \infty} 3^x = +\infty.$$

Our new method of proof in this corrigendum corrects this error and uniform continuity assumption dispensed with.

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